

# Decays of Charmed Vector Mesons — $\eta\pi^0$ mixing as an origin of isospin non-conservation —

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Pion emitting decays and radiative ones of  $D^{*+,0}$  and  $D_s^{*+}$  are studied. As the result, the full-width of  $D^{*0}$  is predicted, by assuming the isospin symmetry. In addition, the isospin non-conserving  $D_s^{*+} \rightarrow D_s^+\pi^0$  decay is investigated under the assumption that it proceeds through the  $\eta\pi^0$  mixing.

It is seen in experiments [1] that the charm strange vector meson  $D_s^{*+}$  decays dominantly into  $D_s^+\gamma$ , while its isospin non-conserving  $D_s^{*+}\pi^0$  decay is much weaker. It is qualitatively understood by a hierarchy of hadron interactions,  $|\textit{isospin conserving ones} \sim O(1)| \gg |\textit{electromagnetic ones} \sim O(\sqrt{\alpha})| \gg |\textit{isospin non-conserving ones} \sim O(\alpha)|$  [2], where  $\alpha$  is the fine structure constant. In reality, it has been observed that this hierarchy plays an important role in decays of charm strange scalar  $D_{s0}^+(2317)$  [3]. In addition, the measured width of  $D^{*+}$  meson has been greatly improved recently [4]. Therefore, we study decays of charmed vector mesons  $D^{*+,0}$  and  $D_s^{*+}$  to see numerically a role of the above hierarchy, in particular, a role of the  $\eta\pi^0$  mixing as an origin of isospin non-conservation.

Using the updated width and branching fractions

$$\begin{aligned} (\Gamma_{D^{*+}})_{\text{exp}} &= 83.4 \pm 1.8 \text{ keV}, & Br(D^{*+} \rightarrow D^0\pi^+)_{\text{exp}} &= 67.7 \pm 0.5 \%, \\ Br(D^{*+} \rightarrow D^+\pi^0)_{\text{exp}} &= 30.7 \pm 0.5 \%, & Br(D^{*+} \rightarrow D^+\gamma)_{\text{exp}} &= 1.6 \pm 0.4 \%, \\ Br(D^{*0} \rightarrow D^0\pi^0)_{\text{exp}} &= 61.9 \pm 2.9 \%, & Br(D^{*0} \rightarrow D^0\gamma)_{\text{exp}} &= 38.1 \pm 2.9 \% \end{aligned} \quad (1)$$

which have been compiled by the Particle Data Group [1], we estimate rates for exclusive decays of  $D^{*+}$  as

$$\begin{aligned} \Gamma(D^{*+} \rightarrow D^0\pi^+)_{\text{exp}} &= 56.5 \pm 1.3 \text{ keV}, & \Gamma(D^{*+} \rightarrow D^+\pi^0)_{\text{exp}} &= 25.6 \pm 0.7 \text{ keV}, \\ \Gamma(D^{*+} \rightarrow D^+\gamma)_{\text{exp}} &= 1.3 \pm 0.4 \text{ keV}. \end{aligned} \quad (2)$$

However, exclusive decay rates of  $D^{*0}$  are not known, because its full-width is not determined yet. Therefore, we estimate them under the isospin  $SU_I(2)$  symmetry, below. Rate for  $D^* \rightarrow D\pi$  decay is given by

$$\Gamma(D^* \rightarrow D\pi) = \frac{|\mathbf{k}_\pi|}{24\pi m_{D^*}^2} \sum_{pol} |M(D^* \rightarrow D\pi)|^2, \quad (3)$$

where  $\mathbf{k}_\pi$  is the center-of-mass (c.m.) momentum of pion in the final state, and the amplitude is written as

$$M(D^*(p) \rightarrow D(p')\pi(k)) = g_{D^*\bar{D}\pi} e_\mu(p) q^\mu, \quad (q = p' - k) \quad (4)$$

in this note. Here,  $g_{D^*\bar{D}\pi}$  and  $e_\mu(p)$  are the  $D^*\bar{D}\pi$  coupling strength and the polarization vector of  $D^*$ , respectively. By using a hard pion technique in the infinite momentum frame (IMF) [5], i.e., by taking  $\mathbf{k} \rightarrow 0$  in the  $\mathbf{p}$  ( $\parallel z$ -axis)  $\rightarrow \infty$  frame, the amplitude is approximated as

$$\begin{aligned} \lim_{\mathbf{p} \rightarrow \infty, \mathbf{k} \rightarrow 0} M(D^* \rightarrow D\pi) &= \left( \frac{m_{D^*}^2 - m_D^2}{f_\pi} \right) \langle D | A_{\bar{\pi}} | D^* \rangle \\ &= -g_{D^*\bar{D}\pi} \left( \frac{m_{D^*}^2 - m_D^2}{m_{D^*}} \right), \end{aligned} \quad (5)$$

under the partially conserved axial-vector current (PCAC) hypothesis [6], where  $A_\pi$  and  $f_\pi$  are the axial-charge with the flavor of pion and the pion decay constant, respectively. Thus, we get

$$g_{D^*\bar{D}\pi} = - \left( \frac{m_{D^*}}{f_\pi} \right) \langle D | A_{\bar{\pi}} | D^* \rangle, \quad (6)$$

which is considered as a meson version of the Goldberger-Treiman relation [7]. We here assume that asymptotic matrix elements of  $A_\pi$  (matrix elements of  $A_\pi$  taken between single hadron states with the infinite momentum),  $\langle D | A_{\bar{\pi}} | D^* \rangle$ 's, satisfy the  $SU_I(2)$  symmetry, i.e.,  $\langle D^+ | A_{\pi^+} | D^{*0} \rangle = 2 \langle D^0 | A_{\pi^0} | D^{*0} \rangle = -2 \langle D^+ | A_{\pi^0} | D^{*+} \rangle = \langle D^0 | A_{\pi^-} | D^{*+} \rangle$ . Under this approximation, we obtain

$$\begin{aligned} \Gamma(D^{*+} \rightarrow D^+\pi^0)_{SU_I(2)} &= 0.463 \Gamma(D^{*+} \rightarrow D^0\pi^+) = 26.1 \pm 0.6 \text{ keV}, \\ \Gamma(D^{*0} \rightarrow D^0\pi^0)_{SU_I(2)} &= 0.580 \Gamma(D^{*+} \rightarrow D^0\pi^+) = 37.1 \pm 1.0 \text{ keV}, \end{aligned} \quad (7)$$

where we have used  $f_{\pi^0} = f_{\pi^\pm}/\sqrt{2}$  and inserted the measured  $\Gamma(D^{*+} \rightarrow D^0\pi^+)_{\text{exp}} = 56.5 \pm 1.3$  keV in Eq. (2) into  $\Gamma(D^{*+} \rightarrow D^0\pi^+)$  in Eq. (7). The estimated  $\Gamma(D^{*+} \rightarrow D^+\pi^0)_{SU_I(2)}$  is consistent with the measured rate in Eq. (2). This implies that the asymptotic  $SU_I(2)$  symmetry ( $SU_I(2)$  symmetry in asymptotic matrix elements) works well in pion emitting strong decays of  $D^*$  mesons. The estimated  $\Gamma(D^{*0} \rightarrow D^0\pi^0)_{SU_I(2)}$  in Eq. (7) and the measured branching fraction  $Br(D^{*0} \rightarrow D^0\pi^0)_{\text{exp}}$  in Eq. (1) lead to the full-width  $(\Gamma_{D^{*0}})_{SU_I(2)} = 59.9 \pm 3.3$  keV of  $D^{*0}$ . Using the above  $(\Gamma_{D^{*0}})_{SU_I(2)}$  and the measured branching fraction  $Br(D^{*0} \rightarrow D^0\gamma)_{\text{exp}}$  in Eq. (1), we obtain

$$\Gamma(D^{*0} \rightarrow D^0\gamma)_{SU_I(2)} = 22.8 \pm 2.2 \text{ keV}. \quad (8)$$

For later convenience, we here list the the following ratio of rates which is obtained from Eqs. (2) and (8),

$$\frac{\Gamma(D^{*0} \rightarrow D^0\gamma)_{SU_I(2)}}{\Gamma(D^{*+} \rightarrow D^+\gamma)_{\text{exp}}} = 17.5 \pm 5.7. \quad (9)$$

Next, we calculate rates for radiative decays of  $D^*$  mesons. They are given in the form

$$\Gamma(D^* \rightarrow D\gamma) = \frac{|\mathbf{k}_\gamma|}{24\pi m_{D^*}^2} \sum_{pol} |M(D^* \rightarrow D\gamma)|^2, \quad (10)$$

where  $\mathbf{k}_\gamma$  is the c.m. momentum of  $\gamma$  in the final state. We here factor out the polarization independent part  $A(D^* \rightarrow D\gamma)$  of the amplitude  $M(D^* \rightarrow D\gamma)$  and write it in the form,

$$A(D^* \rightarrow D\gamma) = \sum_{V=\rho^0, \omega, \phi, \psi} \left[ \frac{X_V(k_\gamma^2=0)}{m_V^2} \right] g_{D^* \bar{D} V}, \quad (11)$$

under the vector meson dominance hypothesis (VMD) [8], where  $X_V(k_\gamma^2=0)$ 's denote the photon-vector meson ( $V = \rho^0, \omega, \phi$  and  $J/\psi$ ) coupling strengths on the photon mass shell. ( $J/\psi$  will be written as  $\psi$  hereafter.) In this manner, the amplitudes for the  $D^{*+ (0)} \rightarrow D^{+ (0)}\gamma$  and  $D_s^{*+} \rightarrow D_s^+\gamma$  decays are explicitly given by

$$A(D^{*+} \rightarrow D^+\gamma) = g_{D^{*+} D^- \rho^0} \frac{X_\rho(0)}{m_\rho^2} + g_{D^{*+} D^- \omega} \frac{X_\omega(0)}{m_\omega^2} + g_{D^{*+} D^- \psi} \frac{X_\psi(0)}{m_\psi^2}, \quad (12)$$

$$A(D^{*0} \rightarrow D^0\gamma) = g_{D^{*0} \bar{D}^0 \rho^0} \frac{X_\rho(0)}{m_\rho^2} + g_{D^{*0} \bar{D}^0 \omega} \frac{X_\omega(0)}{m_\omega^2} + g_{D^{*0} \bar{D}^0 \psi} \frac{X_\psi(0)}{m_\psi^2}, \quad (13)$$

$$A(D_s^{*+} \rightarrow D_s^+\gamma) = g_{D_s^{*+} D_s^- \phi} \frac{X_\phi(0)}{m_\phi^2} + g_{D_s^{*+} D_s^- \psi} \frac{X_\psi(0)}{m_\psi^2}, \quad (14)$$

where it has been assumed that the  $\omega$ - $\phi$ - $\psi$  mixing is ideal and the  $D_{(s)}^* \bar{D}_{(s)} V$  vertices satisfy the OZI rule [9].

To study numerically rates for the above radiative decays, we need to know values of  $X_V(0)$ 's. They can be estimated from values of  $\gamma V$  transition moments ( $\gamma_V$ 's) which are obtained from analyses in atomic number ( $A$ ) dependence of forward cross sections of photoproductions of vector mesons on various targets. The results have been compiled as

$$\begin{aligned} X_\rho(0) &= 0.033 \pm 0.003 \text{ (GeV)}^2, & X_\omega(0) &= 0.011 \pm 0.001 \text{ (GeV)}^2, \\ X_\phi(0) &= -0.018 \pm 0.004 \text{ (GeV)}^2. \end{aligned} \quad (15)$$

in [10] from data on  $\gamma_V$ 's given in the references quoted therein. However, we have updated the value of  $X_\psi(0)$  as

$$X_\psi(0) = 0.15 \pm 0.02 \text{ (GeV)}^2 \quad (16)$$

by using the measured rates for the  $\psi \rightarrow \eta_c \gamma$  and  $\eta_c \rightarrow \gamma \gamma$  decays, because data on forward cross section of  $\psi$  photoproduction seem to be still unstable [11]. Next, it should be recalled [12] that a measure of the flavor symmetry breaking in hadronic interactions is given by the form factor  $f_+(0)$  of related vector-current matrix element at zero momentum transfer squared. Its values have been compiled as [13],  $f_+^{(\pi K)}(0) = 0.961 \pm 0.008$ ,  $f_+^{(\bar{K} D)}(0) = 0.74 \pm 0.03$ ,  $f_+^{(\pi D)}(0)/f_+^{(\bar{K} D)}(0) = 1.00 \pm 0.13$  (FNAL-E687) and  $0.99 \pm 0.08$  (CLEO). These results suggest that the  $SU_f(3)$  symmetry works well (even in the open-charm world), while the  $SU_f(4)$  is broken to an extent of 20 – 30 per cent. Therefore, it is assumed that the  $D_{(s)}^* \bar{D}_{(s)} V$  coupling strengths satisfy the flavor  $SU_f(3)$  symmetry,

$$\sqrt{2}g_{D^{*0} \bar{D}^0 \omega} = \sqrt{2}g_{D^{*+} D^- \omega} = \sqrt{2}g_{D^{*0} \bar{D}^0 \rho^0} = -\sqrt{2}g_{D^{*+} D^- \rho^0} = g_{D_s^{*+} D_s^- \phi}, \quad (17)$$

Table I. Ratios of rates for radiative decays of open-charm vector mesons. The parameter  $x$  is defined in the text.

$x$	1.0	0.9	0.8	0.7	0.6	0.5
$\Gamma(D^{*0} \rightarrow D^0 \gamma)$	42.4	30.6	22.9	17.5	13.7	10.8
$\Gamma(D^{*+} \rightarrow D^+ \gamma)$						
$\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)$	0.0276	0.0780	0.134	0.189	0.241	0.289
$\Gamma(D^{*+} \rightarrow D^+ \gamma)$						

while deviation of  $D_{(s)}^* \bar{D}_{(s)} \psi$  couplings from their  $SU_f(4)$  symmetry limit is parameterized by

$$x = \frac{g_{D^{*+} D^- \psi}}{\sqrt{2} g_{D^{*+} D^- \omega}} = \frac{g_{D^{*0} \bar{D}^0 \psi}}{\sqrt{2} g_{D^{*0} \bar{D}^0 \omega}} = \frac{g_{D_s^{*+} D_s^- \psi}}{g_{D_s^{*+} D_s^- \phi}}, \quad (18)$$

where  $x = 1$  in the  $SU_f(4)$  symmetry limit. In this way, we can give ratios of rates for the above radiative decays by the unknown parameter  $x$ . Their numerical results are listed in Table I. As seen in the table, the ratio of rates in Eq. (9) can be reproduced for  $0.8 \gtrsim x \gtrsim 0.6$ , as expected from the above discussions. When  $x = 0.7$  is taken, our ratio  $\Gamma(D^{*0} \rightarrow D^0 \gamma)_{x=0.7} / \Gamma(D^{*+} \rightarrow D^+ \gamma)_{x=0.7} = 17.5$  agrees to the central value of the phenomenological ratio in Eq. (9), and then

$$\frac{\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)}{\Gamma(D^{*+} \rightarrow D^+ \gamma)} \Big|_{x=0.7} = 0.189 \quad (19)$$

is obtained. Insertion of  $\Gamma(D^{*+} \rightarrow D^+ \gamma)_{\text{exp}} = 1.3 \pm 0.4$  keV in Eq. (2) into the denominator of Eq. (19) leads to

$$\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)_{x=0.7} = (0.25 \pm 0.08) \text{ keV}. \quad (20)$$

From this result, we can estimate the rate for the isospin non-conserving  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay. Because the measured branching fraction has been given by  $Br(D_s^{*+} \rightarrow D_s^+ \gamma)_{\text{exp}} = 94.2 \pm 0.7$  per cent, the full-width of  $D_s^{*+}$  is estimated as  $(\Gamma_{D_s^{*+}})_{x=0.7} = 0.27 \pm 0.09$  keV. Therefore, the rate for the isospin non-conserving  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay is estimated as

$$\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{x=0.7} = 0.016 \pm 0.006 \text{ keV}, \quad (21)$$

because of  $Br(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\text{exp}} = 5.8 \pm 0.7$  per cent. The rates in Eqs. (20) and (21) provide

$$\frac{\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{x=0.7}}{\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)_{x=0.7}} = 0.064 \pm 0.028. \quad (22)$$

This result is consistent with the measured ratio,  $0.62 \pm 0.007$  [1], though our result contains large uncertainties. This implies that the result in Eq. (21) is natural, and therefore, it is compared with our rate  $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\eta\pi^0}$  for the isospin non-conserving decay through the  $\eta\pi^0$  mixing, below.

The charm strange vector meson  $D_s^{*+}$  has no kinematically-allowed hadronic isospin-conserving decay. Its kinematically-allowed decay  $D_s^{*+} \rightarrow D_s^+ \pi^0$  is isospin non-conserving and its rate is given by

$$\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0) = \frac{|\mathbf{k}_{\pi^0}|^3}{6\pi m_{D_s^{*+}}^2} \left| \left( \frac{m_{D_s^{*+}}}{f_{\pi^0}} \right) \langle D_s^+ | A_{\pi^0} | D_s^{*+} \rangle \right|^2 \quad (23)$$

in the same way as the  $D^* \rightarrow D\pi$ , where  $\mathbf{k}_{\pi^0}$  is the c.m. momentum of  $\pi^0$  in the final state and  $f_{\pi^0} = f_{\pi^\pm} / \sqrt{2}$ . We here assume that the decay proceeds through the  $\eta\pi^0$  mixing [14] with the mixing parameter  $\epsilon$ . In this case, the asymptotic matrix element  $\langle D_s^+ | A_{\pi^0} | D_s^{*+} \rangle$  is given by

$$\langle D_s^+ | A_{\pi^0} | D_s^{*+} \rangle = \epsilon \langle D_s^+ | A_\eta | D_s^{*+} \rangle = -\epsilon \sin(\Theta) \langle D_s^+ | A_{\eta_s} | D_s^{*+} \rangle \quad (24)$$

under the asymptotic  $SU_f(3)$  symmetry [5], where  $\Theta = \chi + \theta_P$  with the  $\eta\eta'$  mixing angle  $\theta_P$  and  $\chi = \arccos(\sqrt{1/3}) = \arcsin(\sqrt{2/3}) = 54.7^\circ$ , and  $A_{\eta_s}$  is the axial-charge with the flavor of  $\{\bar{s}s\}$  component of  $\eta$ . The asymptotic  $SU_f(3)$  symmetry implies that the asymptotic matrix elements satisfy  $\langle D_s^+ | A_{\eta_s} | D_s^{*+} \rangle = \langle D^0 | A_{\pi^-} | D^{*+} \rangle$ , so that the following ratio of rates is obtained,

$$R_{\eta\pi^0} = \frac{\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\eta\pi^0}}{\Gamma(D^{*+} \rightarrow D^0 \pi^+)} = \left| \frac{\mathbf{k}_{\pi^0}}{\mathbf{k}_{\pi^+}} \right|^3 \left( \frac{f_\pi}{f_{\pi^0}} \right)^2 [\epsilon \sin(\Theta)]^2. \quad (25)$$

Table II. Rate for the isospin non-conserving  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay through the  $\eta\pi^0$  mixing. In (i), (ii) and (iii), the  $\eta\eta'$  mixing angle is taken as  $\theta_P = -11.4^\circ$ ,  $\theta_P = -24.5^\circ$  and  $\theta_P = -14.1^\circ$ , respectively. The results should be compared with  $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{x=0.7} = 0.016 \pm 0.006$  keV in Eq. (21).

	(i) $\theta_P = -11.4^\circ$	(ii) $\theta_P = -24.5^\circ$	(iii) $\theta_P = -14.1^\circ$
$\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\eta\pi^0}$	$0.0111 \pm 0.0003$ keV	$0.0060 \pm 0.0002$ keV	$0.0100 \pm 0.0003$ keV

Because the  $\eta\eta'$  mixing angle  $\theta_P$  has not been determined yet, we consider the following three cases, (i)  $\theta_P = -11.4^\circ$  (estimated by using the quadratic G-M-O mass formula [15]), (ii)  $\theta_P = -24.5^\circ$  (estimated by using the linear G-M-O mass formula) and (iii)  $\theta_P = -14.1^\circ \pm 2.8^\circ$  (estimated by a lattice QCD simulation [16]), as listed in [1]. In each of these cases, the ratio in Eq. (25) is given by

$$R_{\eta\pi^0} = 1.76|\epsilon|^2 \text{ in (i), } 0.946|\epsilon|^2 \text{ in (ii), } 1.58|\epsilon|^2 \text{ in (iii),} \quad (26)$$

where about 20 per cent errors of the estimated  $\theta_P$  in (iii) have been neglected. Although the mixing parameter  $\epsilon$  was given, long time ago, as  $\epsilon = 0.0105 \pm 0.0013$  [2] which is  $O(\alpha)$  as expected, it is now drastically improved. When we take  $\epsilon = 0.01058$  and  $\Gamma(D^{*+} \rightarrow D^0 \pi^+)_{\text{exp}} = 56.5 \pm 1.3$  keV in Eq. (2) as the input data, we obtain the results listed in Table II. Comparing them with Eq. (21), we find that the cases (i) and (iii) are favored, while the result in (ii) seems to be not favored, though our results involve large uncertainties arising from the  $SU_f(4)$  symmetry breaking parameter  $x$ .

We here compare our results on decays of charmed vector mesons with those from a hybrid model [17]. In this model, pion emitting decays are calculated in a chiral Lagrangian approach in which  $\eta$  participating in the  $\eta\pi^0$  mixing is assumed to be of a pure  $SU_f(3)$ -octet (without any  $\eta\eta'$  mixing) and radiative decays are studied in a constituent quark model. Regarding with the  $SU_I(2)$  symmetry, this model predicted  $\Gamma(D^{*+} \rightarrow D^0 \pi^+)/\Gamma(D^{*+} \rightarrow D^+ \pi^0) \simeq 2$ , in consistency with our result and experiments [1] in Eq. (2). The ratio of rates  $\Gamma(D^{*+} \rightarrow D^+ \gamma)/\Gamma(D^{*0} \rightarrow D^0 \gamma) = 0.058 \pm 0.015$  which was predicted by the hybrid model is accidentally consistent with our estimate in Eq. (9). As for the isospin non-conserving  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay, this theory has predicted  $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\text{hyb}} = 0.0079$  keV, by assuming that the decay is caused by the  $\eta\pi^0$  mixing. Although the value of the mixing parameter is close to ours, the above result is approximately a half of our semi-phenomenological estimate Eq. (21) with which our results in (i) and (iii) are compatible. Regarding with the radiative decay, the hybrid model has predicted  $\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)_{\text{hyb}} = 0.43$  keV by using the constituent quark model. However, this result is larger by about 70 per cent than our estimate Eq. (20) under the VMD. In cosequence, the ratio of rates has been predicted as

$$\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)/\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma) = 0.018 \quad (27)$$

which is far from the measured one  $0.062 \pm 0.008$  [1]. On the other hand, our results on the rate  $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\eta\pi^0}$  in (i) and (iii) reproduce approximately Eq. (21) as seen in Table II, though the same  $\eta\pi^0$  mixing is assumed as the origin of the isospin non-conservation in the  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay, and our estimates of the ratio of rates  $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\eta\pi^0}/\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)_{x=0.7} = 0.044 \pm 0.015$  in (i) and  $0.040 \pm 0.013$  in (iii) are compatible with the measurement, though their errors are still large.

In summary, we have studied decay property of charmed vector mesons, applying a hard pion technique in the IMF (an innovation of the well-known soft pion technique) to pion emitting decays and the VMD to radiative decays. As the result, we have seen that the  $SU_I(2)$  symmetry works well in  $D^* \rightarrow D\pi$  decays and predicted the full-width of  $D^{*0}$  under the  $SU_I(2)$  symmetry. Therefore, measurements of full-width of  $D^{*0}$  are awaited. Then, radiative decays of  $D^*$  and  $D_s^{*+}$  have been studied under the VMD, by assuming that the  $D_{(s)}^* \bar{D}_{(s)} V$ , ( $V = \rho, \omega, \phi$ ) coupling strengths satisfy the  $SU_f(3)$  symmetry, while the deviation from the  $SU_f(4)$  symmetry limit of the  $D_{(s)}^* \bar{D}_{(s)} \psi$  couplings has been considered phenomenologically and has been taken to be 30 percent (i.e.,  $x = 0.7$ ). The isospin non-conserving  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay has been studied by assuming that it proceeds through the  $\eta\pi^0$  mixing. Its rate is explicitly dependent on the  $\eta\eta'$  mixing angle  $\theta_P$ . When  $\theta_P = -11.4^\circ$  (from the quadratic G-M-O mass formula) and  $\theta_P = -14.1^\circ$  (from a lattice QCD simulation) have been taken, our values of ratio of rates  $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)_{\eta\pi^0}/\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)_{x=0.7}$  have been compatible with the measured one, while the result with  $\theta_P = -24.5^\circ$  (from the linear G-M-O mass formula) has not been favored by the measured ratio. Therefore, determinations of the  $\eta\eta'$  mixing angle will be important to establish the role of the  $\eta\pi^0$  mixing as the origin of the isospin non-conservation in the  $D_s^{*+} \rightarrow D_s^+ \pi^0$  decay.

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